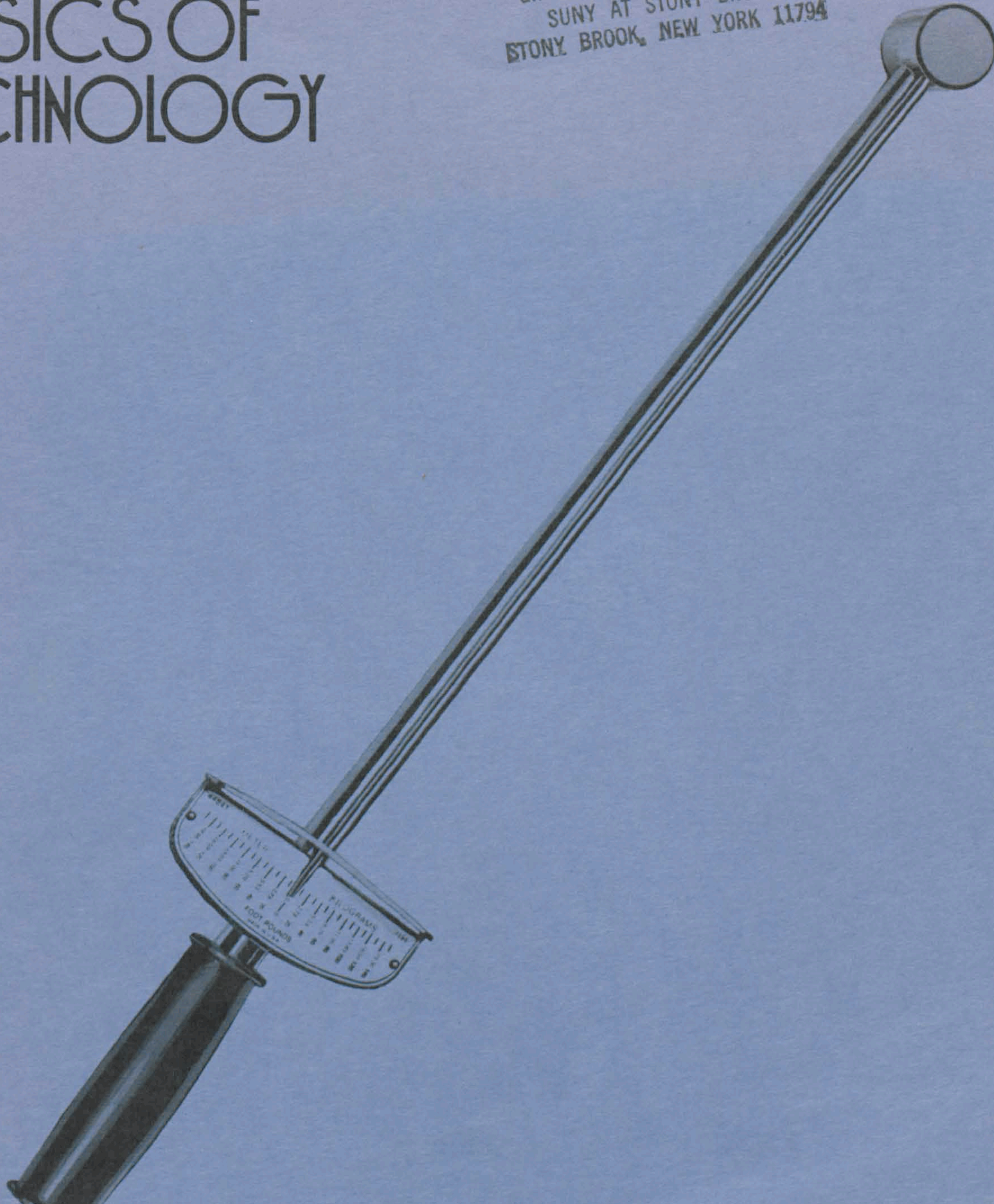
pt SUNY

PHYSICS OF TECHNOLOGY



COORDINATED BY AMERICAN INSTITUTE OF PHYSICS

THE TORQUE WRENCH

Forces, Torques, and Elasticity

THE TORQUE WRENCH

A Module on Forces, Torques, and Elasticity

SUNY at Binghamton

Bruce B. Marsh, State University of New York at Albany

Carl R. Stannard, State University of New York at Binghamton
Bruce B. Marsh, State University of New York at Albany
Project Co-Directors

McGRAW-HILL BOOK COMPANY

NEW YORK
ST. LOUIS
DALLAS
SAN FRANCISCO
AUCKLAND
DÜSSELDORF
JOHANNESBURG
KUALA LUMPUR
LONDON
MEXICO
MONTREAL
NEW DELHI
PANAMA
PARIS
SÃO PAULO
SINGAPORE
SYDNEY
TOKYO
TORONTO

The Physics of Technology modules were produced by the Tech Physics Project, which was funded by grants from the National Science Foundation. The work was coordinated by the American Institute of Physics. In the early planning stages, the Tech Physics Project received a small grant for exploratory work from the Exxon Educational Foundation.

The modules were coordinated, edited, and printed copy produced by the staff at Indiana State University at Terre Haute. The staff involved in the project included:

Philip DiLavore . . . . Editor
Julius Sigler . . . . Rewrite Editor
Mary Lu McFall . . . . Copy and Layout Editor
B. W. Barricklow . . . . Illustrator
Stacy Garrett . . . . Compositor
Elsie Green. . . . . Compositor
Donald Emmons . . . . Technical Proofreader

In the early days of the Tech Physics Project A. A. Strassenburg, then Director of the AIP Office of Education, coordinated the module quality-control and advisory functions of the National Steering Committee. In 1972 Philip DiLavore became Project Coordinator and also assumed the responsibilities of editing and producing the final page copy for the modules.

The National Steering Committee appointed by the American Institute of Physics has played an important role in the development and review of these modules. Members of this committee are:

J. David Gavenda, Chairman, University of Texas, Austin
D. Murray Alexander, DeAnza College
Lewis Fibel, Virginia Polytechnic Institute & State University
Kenneth Ford, New Mexico Institute of Mining and Technology
James Heinselman, Los Angeles City College
Alan Holden, Bell Telephone Labs
George Kesler, Engineering Consultant
Theodore Pohrte, Dallas County Community College District
Charles Shoup, Cabot Corporation
Louis Wertman, New York City Community College

This module was written and tested at the State University of New York at Binghamton. In addition to the Project Co-Directors, the professional staff includes Arnold Benton, the American Institute of Physics; Malcolm Goldberg, Westchester Community College; Giovanni Impeduglia, Staten Island Community College; Gabriel Kousourou, Queensborough Community College; Ludwig P. Lange, Broome Community College, John F. Ouderkirk, SUNY ATC, Canton; and Arnold A. Strassenburg, SUNY, Stony Brook.

The authors wish to express their appreciation for the help of many people in bringing this module to final form. The criticisms of various reviewers and the cooperation of field-test teachers have been most helpful. Several members of the staff of the State University of New York at Binghamton also deserve special recognition for their contributions. They are: Felix K. Chen, Duane A. Damiano, Eugene C. Deci, Barry R. DeGraff, Max Eibert, Steve I. Finette, Chung-Yu Fu, Jeffrey W. Gray, Leonard R. Kelly, Stevan H. Leiden, Debra M. Simes, Laurie A. Sperling, William Standish, and Howard M. Zendle.

The Torque Wrench

ISBN 0-07-001725-5

2 3 4 5 6 7 8 9 0 E B E B 7 8 3 2 1 0 9 8 7 6 5

TABLE OF CONTENTS

The Torque Wrench

GOALS OF THE MODULE

This list of goals indicates what you should be able to do *after* completing the study of this module.

1. Define clearly the following: torque, torque wrench, lever arm, equilibrium, static equilibrium, spring constant, cantilever beam.

2. Given the magnitude and direction of a force and its point of application on an object, determine the lever arm and the torque about a specified axis.

3. State the conditions which the applied forces and torques must satisfy for an object to be in equilibrium.

4. Be able to use the ideas of torque and equilibrium to solve simple problems involving static equilibrium.

5. State Hooke's law.

6. Given empirical information in either tabular or graphical form on how a system responds to an applied force, determine if it obeys Hooke's law. If so, determine the spring constant.

7. Be able to describe qualitatively how Hooke's law depends on system parameters, such as thickness and length, etc.

8. Be able to apply Hooke's law to more complicated systems, such as a compressed or stretched rod and a cantilever beam.

INTRODUCTION

Wrenches come in a large variety of sizes and shapes, as in Figure 1. Despite the differences in appearance, all wrenches have two things in common:

1. They are designed to rotate the objects to which they are applied; that is, they are made to produce a *torque*.
2. They flex while being used. (For most wrenches the flexing is too small to be noticeable.)

Both of these characteristics will be explored in this module by studying a special kind of wrench called a *torque wrench*. The principles associated with the design and operation of the torque wrench are fundamental to many areas of science and technology. The goal of this module is to help you become familiar with these principles. Specifically, you will learn about forces and torques, and you will study some mechanical properties of the materials of which wrenches are made.

You may wonder why a torque wrench would be used instead of a more common type of wrench. The answer is simple. In some applications it is important that sets of bolts be tightened uniformly. The set of bolts (called *head* bolts) which attach the head of an automobile engine to the block is a good example. The technical information for an engine specifies the torque to which the head bolts are to be tightened. The specified torque is chosen to ensure that the bolts are tight enough to securely hold the head in position, and yet not so tight that the forces developed when the engine is running would cause the bolts or the head to crack.

A similar application of torque wrenches is to seal high-vacuum systems which use copper gaskets. In order to insure uniform crushing of the gasket and thus good seal, both the torque and tightening sequence are specified.

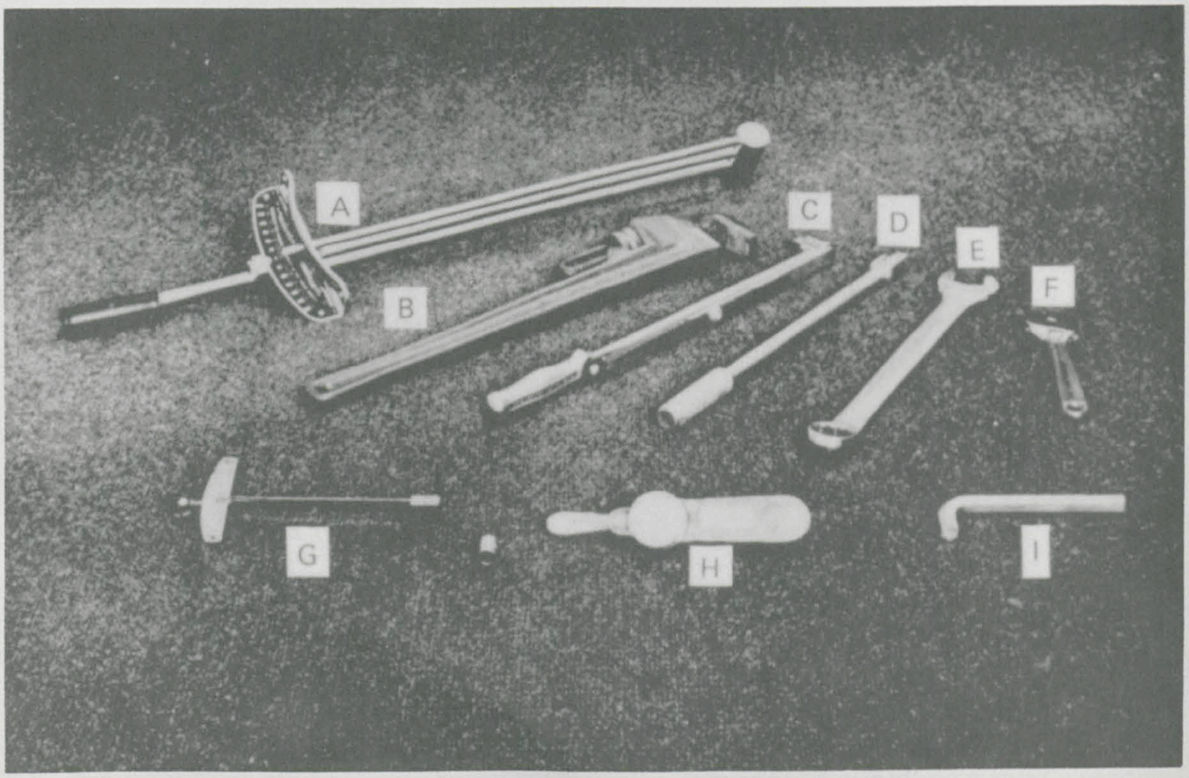


Figure 1. The wrenches shown are of the following types: A, C, G, and H, various kinds of torque wrenches; B, pipe wrench; D, socket wrench; E, combination wrench; F, adjustable (crescent) wrench; I, Allen wrench.

Let's consider the first point of similarity among wrenches, their ability to produce torque. The torque wrench, the box wrench, the pipe wrench, the socket wrench, the crescent wrench, and the Allen wrench don't look alike, yet each of them is used to develop a torque (twisting effect). They differ in how they grip, and in the shape of the handle. In each case a force applied to the handle produces a twist on a nut or bolt. Since each wrench produces a torque, more proper names might be "box torque wrench" or "pipe torque wrench." Why do we reserve the name "torque wrench" for only one member of the wrench family? Perhaps it should be called a "torque measuring wrench," since it *measures* torque.

The ability of a torque wrench to measure torque is a result of the second characteristic shared by wrenches. The handles of all wrenches flex. Most wrenches have such strong handles that the flexing isn't noticed. However, the torque wrench makes use of the substantial flexing in the handle. Torque wrenches have scales and pointers to indicate the amount of torque. Figure 2 shows a commonly used torque wrench and its scale. The handle of the torque wrench flexes when a force is applied, then returns to its original shape when the force is removed. The property of the metal in the wrench that causes the wrench to return to its original shape after being distorted is called *elasticity*. This elastic property allows the applied torque to be measured. Thus, a mechanic can know when he has tightened a bolt to the manufacturer's specification.

In summary, all wrenches *produce torque* and all *flex* elastically when used. The torque wrench is unique in that the amount of torque it produces is shown by a built-in scale. These features will be used to study torque, equilibrium, and elasticity in subsequent sections of this module.

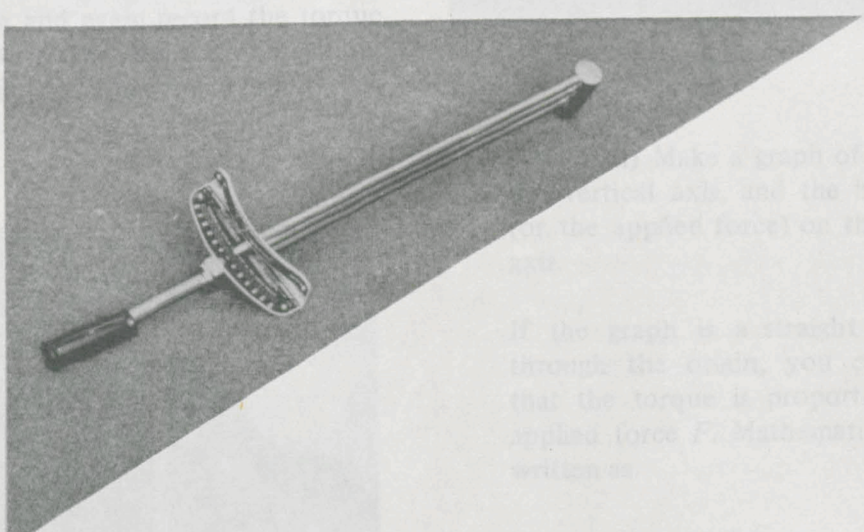


Figure 2. A commonly used torque wrench calibrated in ft·lb.

SECTION A

Torque and Equilibrium

INTRODUCTION

Torque is the technical name for the twisting effect produced by a wrench. Although wrenches aren't the only devices that produce torque, we'll concentrate on them for the moment. The first experiment is designed to show how the torque depends on three factors:

1. The force applied to the handle

2. The distance from the axis of rotation to the point of application of the force

3. The angle between the handle and the line along which the force acts.

Knowledge of the relation between torque and these factors will allow us to define torque precisely. For the moment, you may define torque as that which is measured by a torque wrench. Such a definition is called an *operational definition*.

EXPERIMENT A-1. Torque

A. Changing the Amount of the Hanging Mass

1. Hang a mass from the end of a lightweight support (called a *load arm*) as shown in Figure 3. Use a screwdriver to get a feeling for the amount of torque required to hold the arm horizontal. To measure the torque, use the torque wrench, as shown in Figure 4. The scale on the wrench indicates the amount of torque applied by the wrench to the axis of the load arm. When the load arm is held in the horizontal position, this applied torque just "balances" that produced by the hanging mass. The two torques are equal and opposite. Thus you can record the torque measured by the wrench as the torque produced by the hanging mass.

2. Suspend a different mass from the end of the arm and again record the torque to hold it in the horizontal position. Repeat this procedure for several different masses.

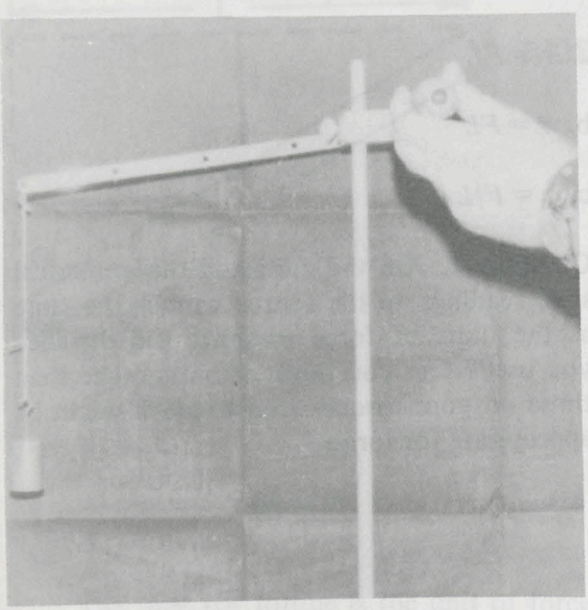


Figure 3.

In each case, the torque you measure is produced by a force, the *weight* of the suspended mass. The weight is proportional to the mass. (If you wish to convert mass units to force units, see the Appendix at the end of the module.)

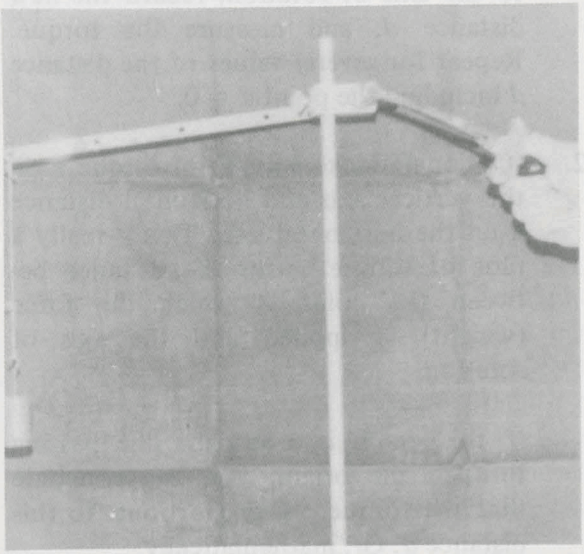


Figure 4.

3. (Optional) Make a graph of torque τ on the vertical axis, and the hanging mass (or the applied force) on the horizontal axis.

If the graph is a straight line passing through the origin, you can conclude that the torque is proportional to the applied force F. Mathematically, this is written as

$$\tau \propto F$$

B. Changing the Position of the Hanging Mass

1. Does the point from which the mass is suspended affect the amount of torque produced? To find out, repeat this experiment. However, this time measure the torques required to hold the arm

horizontal as the *same* mass is suspended at different points along the arm. First, suspend a mass from the end of the arm and measure the distance d from the point of suspension to the axis about which the load arm rotates. Record the torque required to hold the load arm horizontal. Move the hanging mass closer to the axis of rotation, record the new distance d, and measure the torque. Repeat for several values of the distance d including the point $d = 0$.

2. (Optional) Make a graph of torque τ on the vertical axis and horizontal distance d on the horizontal axis. This is really a plot of torque versus the distance between the point at which the force (weight) is applied and the axis of rotation.

   If this graph is a straight line passing through the origin, you can conclude that the torque τ is proportional to this distance d. Or, mathematically

$$\tau \propto d$$

C. Changing the Angle of the Lever Arm

What happens if the arm is not horizontal? Before making further measurements, some new terms are needed. In Figure 5 the vertical line is an extension of the line along which the force acts. The line is called the *line of action* of the applied force. The line labelled "L" in the Figure is the perpendicular from the axis to the line of action of the force. It is called the *lever arm*.

Hang the same mass as in part B from the end of the load arm. You can now vary the length of the lever arm by changing the angle the load arm makes with the horizontal.

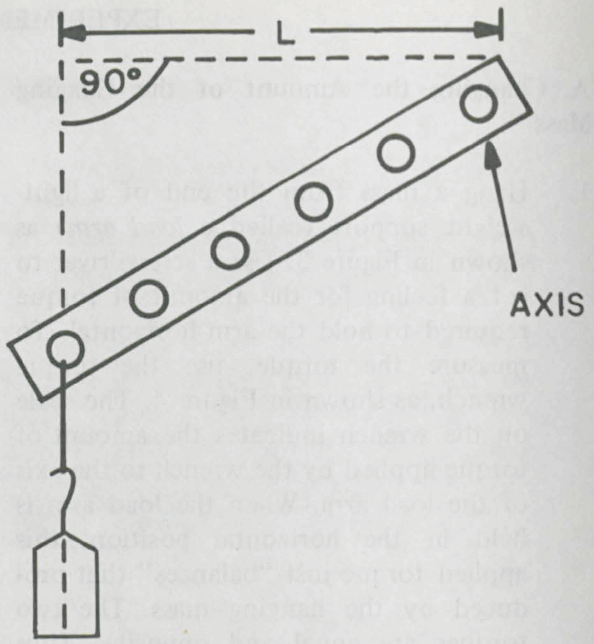


Figure 5.

Choose lengths of the lever arm which are the same as the distances d measured in part B. Measure and record the torque required to hold the load arm at each position. How do these values of torque compare to those measured in part B?

Question 1. Which, if any, of the equations below is consistent with *all* of the observations made in this experiment?

a. $\tau = FL$

b. $\tau = FL^2$

c. $\tau = F/L$

Question 2. Are the units and the numerical scale readings on the torque wrench the same as the units of force (weight) and distance you used? Can you think of units other than those on your torque wrench which might be appropriate for torque?

THE DEFINITION OF TORQUE

It is consistent with the results of Experiment A-1 to define torque by means of the equation

$$\tau = F \cdot L \qquad (1)$$

The torque wrench which you used is calibrated to agree with this definition.

Sometimes it is more convenient to express the torque in terms of the actual distance from an axis of rotation to the point at which the force is applied. That distance is labelled r in Figure 6. The angle between r and F is called θ. For the right triangle shown in Figure 6, the sine of θ is given by

$$\sin\theta = L/r \qquad (2)$$

Thus,

$$L = r\sin\theta$$

and

$$\tau = Fr\sin\theta \qquad (3)$$

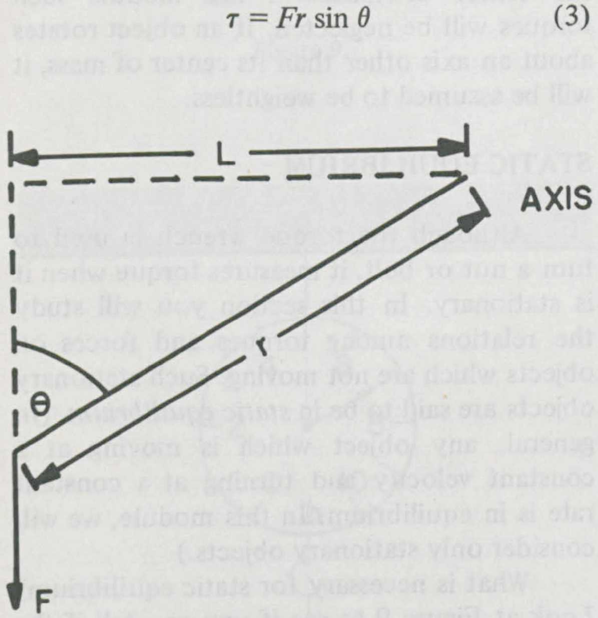


Figure 6.

The unit for torque is the unit of force times the unit of length. The most common units in use in this country are pound-foot (lb·ft), pound-inch (lb·in), and ounce-inch (oz·in). Most other countries, and many scientists in this country, use the metric system, in which the unit for torque is newton-meter (N·m).

Example 1. The length (r) of the wrench in Figure 7 is 12 in and the applied force is 10 lb. Determine the torque applied to the nut for:

a. $\theta = 90°$

b. $\theta = 30°$

c. $\theta = 0°$

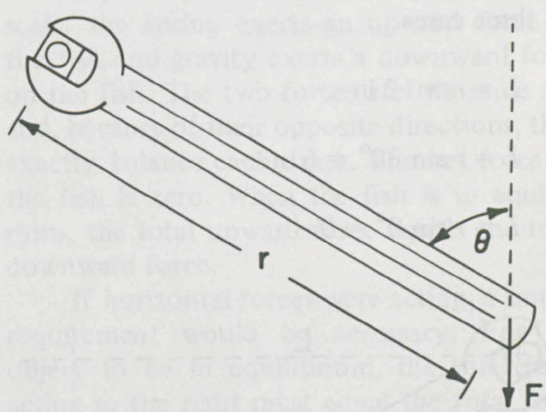


Figure 7.

Solution. The torque is obtained by use of Equation (3).

$$\tau = Fr\sin\theta$$

$$= (10\text{ lb})\,(12\text{ in})\,(\sin\theta)$$

$$= (10\text{ lb})\,(1\text{ ft})\,(\sin\theta)$$

Inserting the different values of θ:

a. $\tau = (10\text{ lb})\,(1\text{ ft})\,(\sin 90°)$

$= 10\text{ lb}\cdot\text{ft}$

b. $\tau = (10 \text{ lb})\,(1 \text{ ft})\,(\sin 30°)$

$= (10 \text{ lb})\,(1 \text{ ft})\,(0.5)$

$= 5 \text{ lb}\cdot\text{ft}$

c. $\tau = (10 \text{ lb})\,(1 \text{ ft})\,(\sin 0°)$

$= (10 \text{ lb})\,(1 \text{ ft})\,(0)$

$= 0$

As the angle θ decreases from 90° the torque decreases even though the applied force remains the same. This happens because the lever arm L shown in Figure 8 decreases with decreasing angle. The perpendicular distance from the axis to the line of action of the force (the lever arm) has these values for the three cases.

a. $L = r = 12$ in

b. $L = r \sin 30° = 6$ in

c. $L = r \sin 0° = 0$

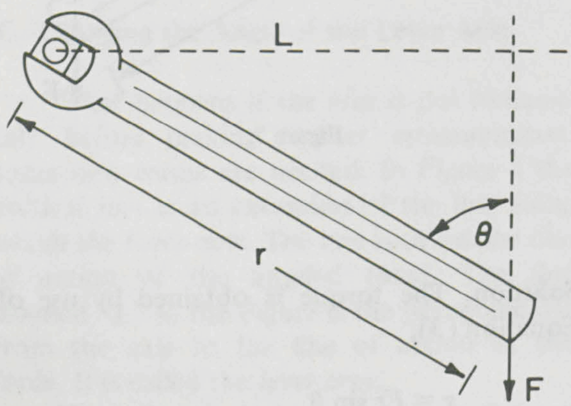


Figure 8.

Question 3. Do any of the measurements made in Experiment A-1 confirm the answer to c?

Problem 1. In a situation such as Example 1, if it is necessary to apply a torque of 100 lb·in with a force of 8 lb when $\theta = 60°$, how long must the wrench handle be?

In Experiment A-1 you probably noticed that, even with no weight attached, some torque is necessary to hold the load arm in a horizontal position. This is not surprising since the load arm has some weight which, by our definition, must exert a torque. However, each particle in the arm has mass (weight) and exerts a torque equal to the product of its weight and its lever arm. Since we cannot easily specify the position of each particle which comprises the load arm, we take some kind of "average" value of the positions of all the particles. We call this average position the *center of mass*.

The center of mass of an object is a single point at which all of its mass is considered to be concentrated. For many objects, the center of mass is at the center. For other objects, the location of the center of mass may be difficult to compute, but it can be located experimentally.

For an axis which does not pass through the center of mass, the weight of the object exerts a torque equal to the product of the weight and the lever arm between the axis and the center of mass. In this module such torques will be neglected. If an object rotates about an axis other than its center of mass, it will be assumed to be weightless.

STATIC EQUILIBRIUM

Although the torque wrench is used to turn a nut or bolt, it measures torque when it is stationary. In this section you will study the relations among torques and forces on objects which are not moving. Such stationary objects are said to be in *static equilibrium*. (In general, any object which is moving at a constant velocity and turning at a constant rate is in equilibrium. In this module, we will consider only stationary objects.)

What is necessary for static equilibrium? Look at Figure 9 to see if you can tell if the situation shown is in equilibrium.

Experience tells us that the fish and scale are stationary. There is a downward force of, say 100 lb, acting on the fish (its weight). There is an upward force of 100 lb, also acting on the fish, and exerted by the scale. The force exerted by the scale on the fish and

the weight of the fish act through a single point. Forces that act through a single point are called *concurrent forces*.

Figure 10 shows a situation in which forces do not act through a single point. These forces are called *nonconcurrent*. The arrangement shown in Figure 10 will balance.

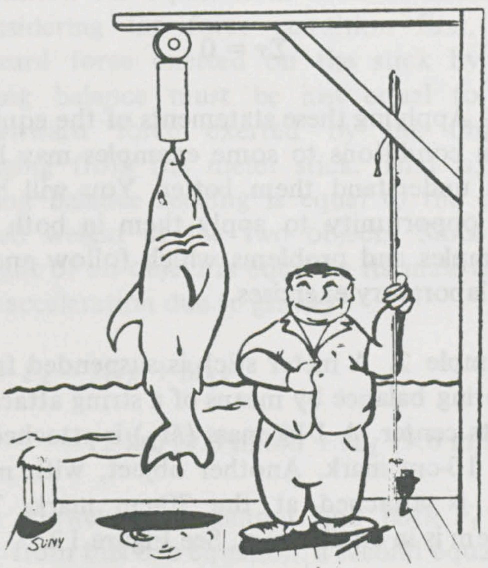


Figure 9.

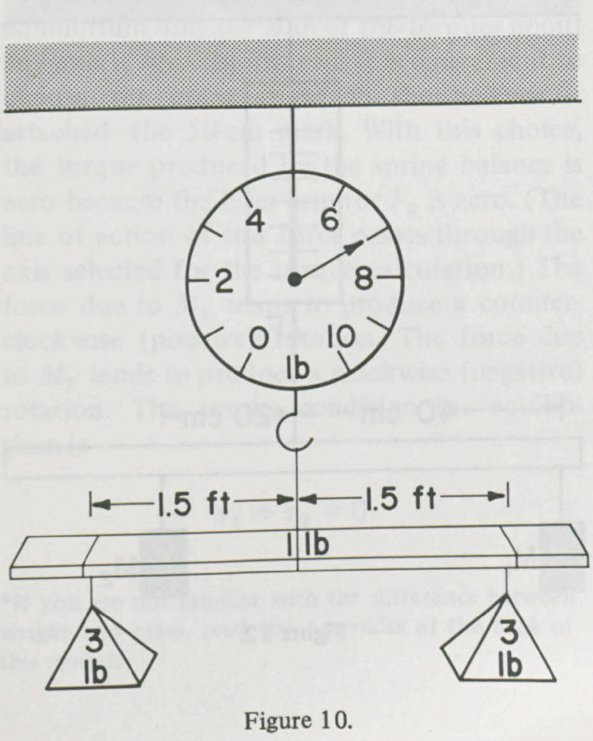


Figure 10.

If the bar weighs 1 lb, is the reading on the scale what one might expect? The static equilibrium of this situation is upset by moving either weight along the bar. If that is done, what kind of motion results? Do the weights produce torques about the point at which the bar is suspended?

In thinking about the situations described above, you may have realized that for an object to be in equilibrium, the forces acting on the object must somehow cancel. If the net force on an object is not zero, the object will be accelerated. Also, the torques acting on the object must cancel. If the net torque on the object is not zero, the object will rotate. We will find a mathematical way of stating what is meant when we say that forces and torques cancel.

In the case of the fish hanging from the scale, the spring exerts an upward force on the fish and gravity exerts a downward force on the fish. The two forces are the same size and, because of their opposite directions, they exactly balance each other. The net force on the fish is zero. When the fish is in equilibrium, the total upward force equals the total downward force.

If horizontal forces were acting, a similar requirement would be necessary. For the object to be in equilibrium, the total force acting to the right must equal the total force acting to the left.

Although describing forces in terms of "up-down" and "right-left" is good enough for many situations, it is often necessary to have a more precise way to refer to directions. This is done by introducing a set of *coordinate axes*. In Figure 11 the x-axis and the y-axis are perpendicular to each other. They provide a convenient way to label directions. (Sometimes it is convenient to define a third (z) axis which is perpendicular to each of the other two.)

Now we can state that a body is in equilibrium if the total force acting in the positive x-direction is the same as that acting in the negative x-direction. The same conditions must hold for the y- and z-directions.

We arbitrarily agree to call a force along an axis positive if it acts in the positive direction of the axis and negative if it acts in

the negative direction of the axis. With this choice, it is possible to restate the force condition for equilibrium. For an object to be in equilibrium, the sum of the forces in the x-direction must equal zero (that is, combining positive and negative numbers), and likewise for y and z. It is usually stated in somewhat briefer form: *for an object to be in equilibrium, the net force in any direction is zero.*

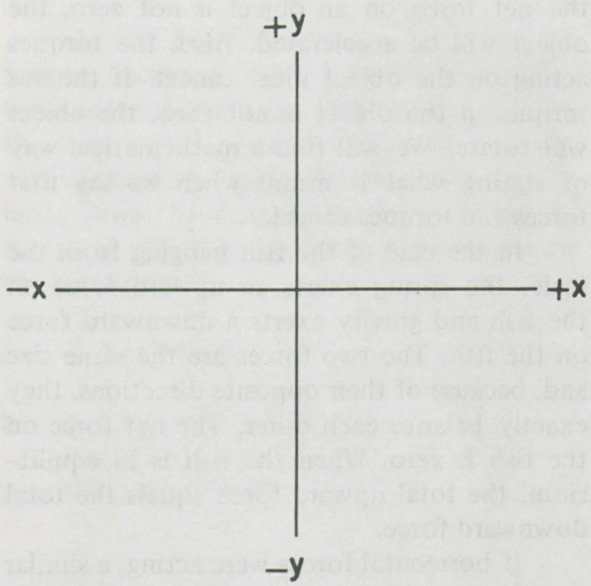


Figure 11.

This statement is often given in the form of three equations

$$\Sigma F_x = 0$$

$$\Sigma F_y = 0$$

$$\Sigma F_z = 0 \tag{4}$$

The Greek letter *sigma* (Σ) is an instruction to add together all of the forces. Thus, ΣF_x means "add algebraically all of the forces in the x-direction."

A similar statement is true for torques. Torques about a given axis tend to produce either a clockwise or a counterclockwise rotation about that axis. For an object to be in equilibrium, the total clockwise torque must equal the total counterclockwise torque about *any* arbitrary axis. (It is customary to label counterclockwise torques positive and clockwise torques negative.) Then, the torque condition for equilibrium is as follows: *for an object to be in equilibrium the net torque about any axis is zero*. Mathematically, we can write

$$\Sigma \tau = 0 \tag{5}$$

Applying these statements of the equilibrium conditions to some examples may help you understand them better. You will have the opportunity to apply them in both the examples and problems which follow and in the laboratory exercises.

Example 2. A meter stick is suspended from a spring balance by means of a string attached to its center. A 1-kg mass (M_1) is attached at the 10-cm mark. Another object, with mass M_2, is attached at the 70-cm mark. The system is in equilibrium. See Figure 12.

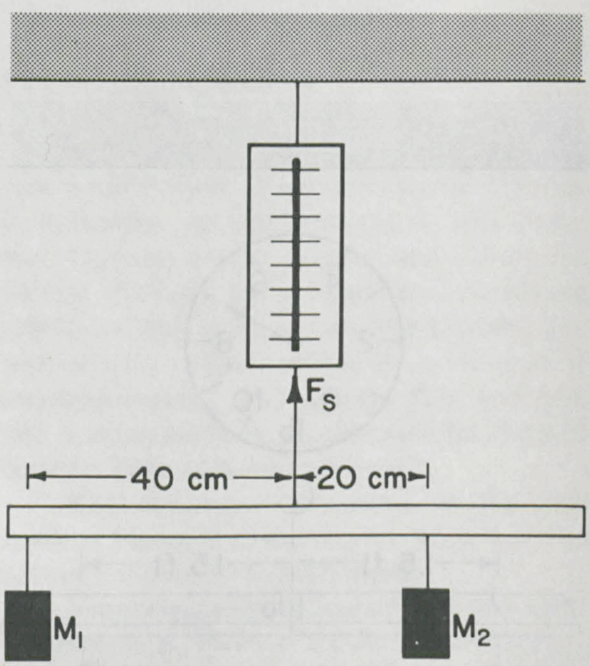


Figure 12.

a. What is the value of M_2?

b. What is the reading on the spring balance?

Assume that the weight of the meter stick is so small it can be ignored.

Solution. Both the force and the torque condition for equilibrium must be satisfied. Considering the force condition first, the upward force exerted on the stick by the spring balance must be just equal to the downward force exerted by the objects hanging from the meter stick. That is, the spring balance reading is equal to the combined weight of the two objects. Since the weight of an object is equal to its mass times the acceleration due to gravity*

$$W = F_s = M_1 g + M_2 g$$

$$= (1 \text{ kg})\,(9.8 \text{ m/s}^2) + M_2\ (9.8 \text{ m/s}^2)$$

However, we cannot find both F_s and M_2 from this one equation; a second equation is needed. That second equation follows from the torque condition. First, we must select an axis about which to compute the torques. We can choose *any* axis, because the object is in equilibrium and the sum of the torques about any axis is zero. In this case it is convenient to choose the point at which the balance is attached–the 50-cm mark. With this choice, the torque produced by the spring balance is zero because the lever arm for F_s is zero. (The line of action of this force passes through the axis selected for the torque calculation.) The force due to M_1 tends to produce a counterclockwise (positive) rotation. The force due to M_2 tends to produce a clockwise (negative) rotation. The torque condition for equilibrium is

$$\tau_1 + \tau_2 = 0$$

*If you are not familiar with the difference between weight and mass, read the Appendix at the back of this module.

Using the appropriate sign convention, this becomes

$$F_1 L_1 - F_2 L_2 = 0$$

where $L_1 = 40$ cm and $L_2 = 20$ cm. Since the forces are due to the weights of masses, $F_1 = M_1 g$ and $F_2 = M_2 g$, so that

$$M_1 g L_1 - M_2 g L_2 = 0$$

This can be solved for M_2:

$$M_2 = \frac{M_1 g L_1}{g L_2} = \frac{M_1 L_1}{L_2}$$

$$= (1 \text{ kg})\,(40 \text{ cm})/(20 \text{ cm}) = 2 \text{ kg}$$

Then, using the equation obtained from the force condition the scale reading is

$$F_s = M_1 g + M_2 g = (M_1 + M_2) g$$

$$= (1 \text{ kg} + 2 \text{ kg})\,(9.8 \text{ m/s}^2)$$

$$= 29.4 \text{ N}$$

To emphasize the point that, in equilibrium, the total torque about any axis is zero, suppose we compute F_s by using the torque about the left end of the meter stick. There are two clockwise (negative) torques due to the hanging masses M_1 and M_2, and one counterclockwise (positive) torque due to the force exerted by the spring on the meter stick. The sum of these three torques must be zero:

$$-M_1 g L_1 - M_2 g L_2 + F_s L_3 = 0$$

Now, $L_1 = 10$ cm, $L_2 = 70$ cm and $L_3 = 50$ cm. Solving for F_s:

$$F_s = (M_1 L_1 + M_2 L_2)\frac{g}{L_3}$$

$$= (1 \text{ kg} \times 10 \text{ cm} + 2 \text{ kg} \times 70 \text{ cm})\,\frac{9.8 \text{ m/s}^2}{50 \text{ cm}}$$

$$= 29.4 \text{ N}$$

Example 3. Figure 13 shows an oddly shaped object hanging in equilibrium from a spring balance. Its center of mass is at the point indicated and a force $F_2 = 19.6$ N is applied to keep it balanced. What is the object's mass?

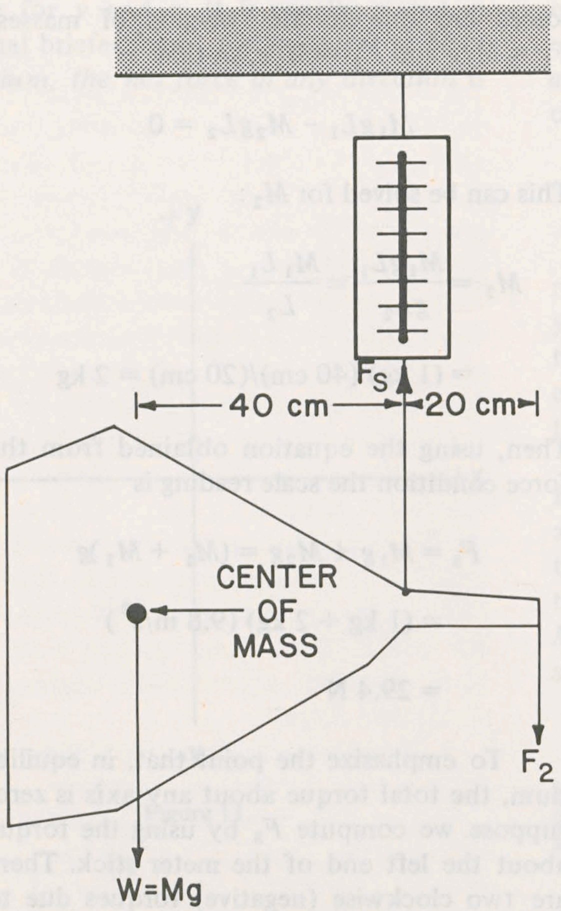


Figure 13.

Solution. This problem is similar to Example 2. The force F_2 is the same and, since the weight behaves as if it were concentrated at the center of mass, the lever arms are also the same as those for the preceding example. Using the torque condition

$$F_1 L_1 - F_2 L_2 = 0$$

$$MgL_1 - F_2 L_2 = 0$$

Solving for M:

$$M = \frac{F_2 L_2}{gL_1} = \frac{19.6 \text{ N} \times 20 \text{ cm}}{9.8 \text{ m/s}^2 \times 40 \text{ cm}}$$

$$= 1 \text{ kg}$$

As we might expect, the mass of the object is the same as M_1 in Example 2. Use of the force condition will show that the reading of the spring balance is also the same as that for the previous example.

Problem 2. For the equilibrium situation shown in Figure 14, determine the values of M and D. Ignore the weight of the stick.

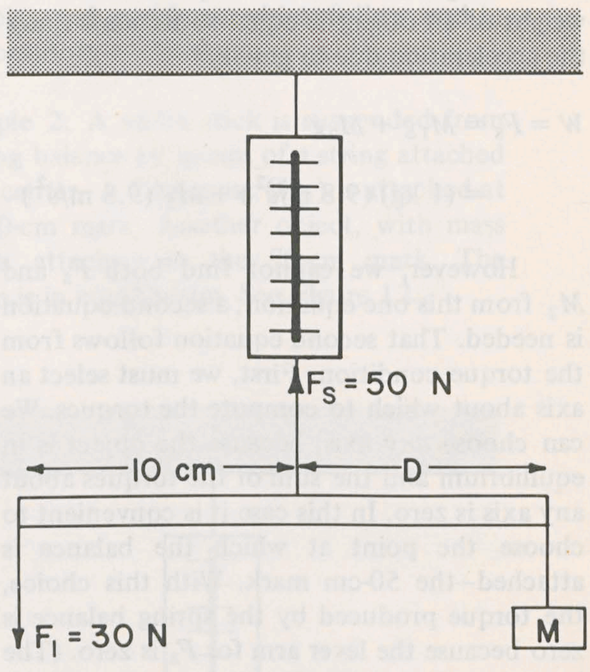


Figure 14.

Example 4. For Figure 15 determine the value of F_1 needed to produce equilibrium. Ignore the weight of the stick.

Solution. The torque about the point of attachment of the spring due to F_2 is clockwise (negative). Its magnitude can be calculated from Equations (2) or (3).

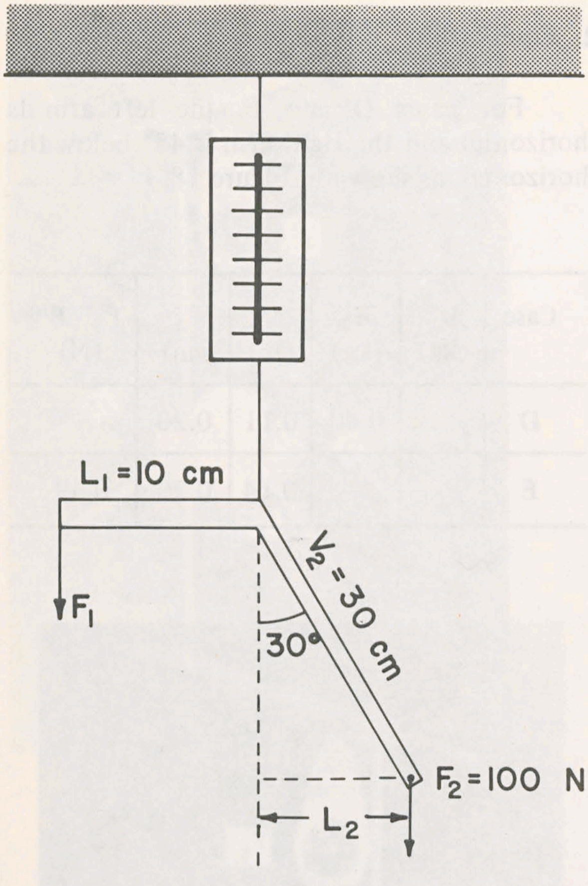


Figure 15.

$$\tau_2 = -F_2 L_2$$

$$= -F_2 r_2 \sin \theta_2$$

$$= -(100 \text{ N})\ (0.3 \text{ m})\ (0.5)$$

$$= -15 \text{ N}\cdot\text{m}$$

For equilibrium, F_1 must produce a torque of the same magnitude in the counter-clockwise direction. That is

$$\tau_1 = -\tau_2$$

$$\tau_1 = F_1\ (0.1 \text{ m}) = -(-15 \text{ N}\cdot\text{m})$$

$$F_1 = (15 \text{ N}\cdot\text{m})/(0.1 \text{ m}) = 150 \text{ N}$$

Problem 3. For the equilibrium situation shown in Figure 16, determine the value of F_1.

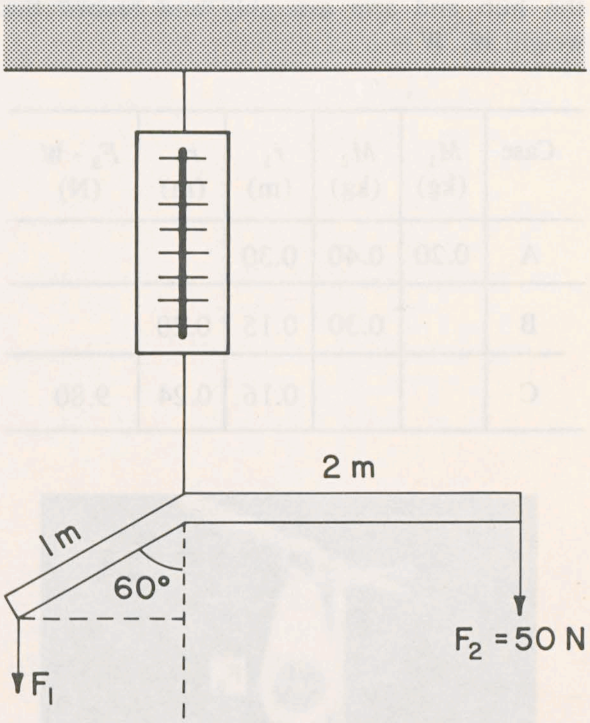


Figure 16.

EXPERIMENT A-2. Equilibrium

For each of the situations described, *predict* the equilibrium values of the unknowns (indicated by blanks in the tables), then verify these values by using the experimental apparatus. Your predictions should be based on the two conditions for equilibrium: the net force equals zero and the net torque equals zero. Enter your prediction in the top half of each box and the observed value in the bottom half.

For cases A, B, and C, both arms are horizontal as shown in Figure 17. First weigh the hub and arm assembly and record the weight W. $W =$ ______

Case	M_1 (kg)	M_2 (kg)	r_1 (m)	r_2 (m)	$F_s - W$ (N)
A	0.20	0.40	0.30		
B		0.30	0.15	0.20	
C			0.16	0.24	9.80

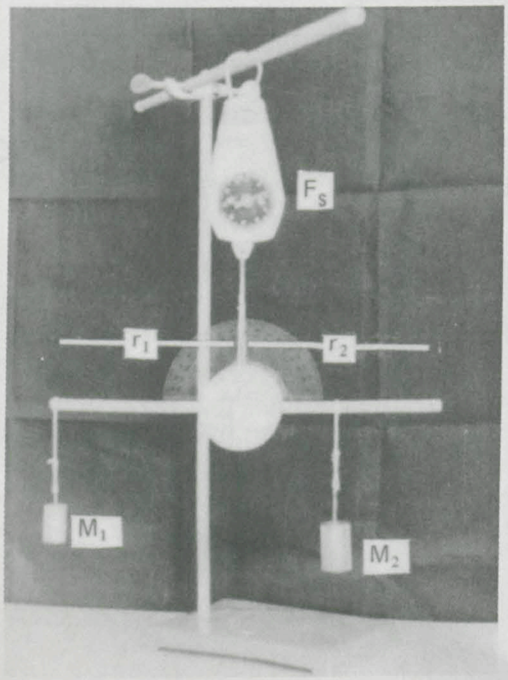


Figure 17.

For cases D and E, the left arm is horizontal and the right arm is 45° below the horizontal, as shown in Figure 18. $W =$ ______

Case	M_1 (kg)	M_2 (kg)	r_1 (m)	r_2 (m)	$F_s - W$ (N)
D		0.40	0.11	0.20	
E			0.14	0.20	0.49

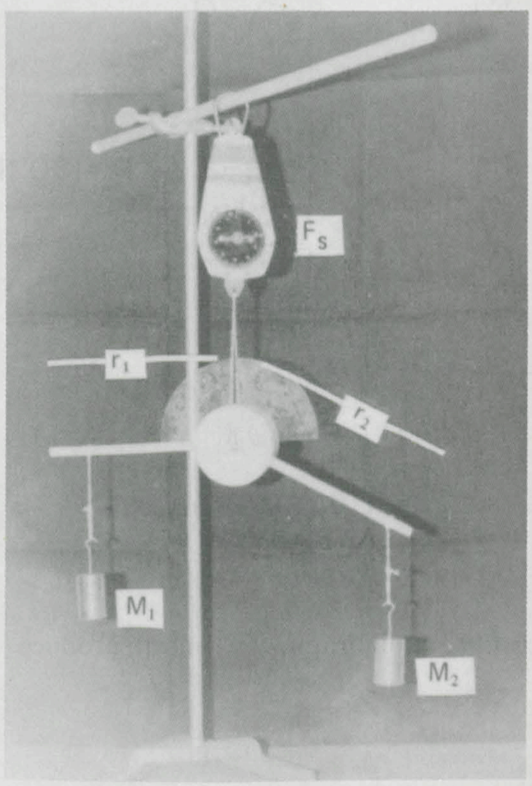


Figure 18.

Case	M_1	M_2	r_1	r_2
F	0.50		0.10	0.33

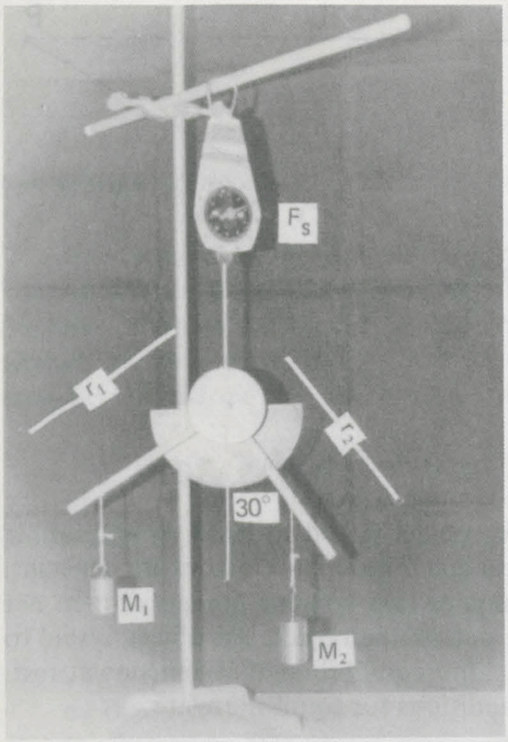


Figure 19.

For case F the arms form a right angle, as in Figure 19. The right arm makes an angle of 30° with the vertical. $W =$ ______

Each of the preceding examples, as well as the experiments, have involved forces in the vertical direction only. If horizontal forces are also present, the equilibrium conditions demand that the net horizontal force also be zero. More complicated equilibrium problems are easily solved if one uses each of the equilibrium conditions in turn. The algebra may become complicated, but the principle is precisely the same. The net force along any direction is zero, and the net torque about any axis is zero.

SUMMARY

The twisting effect (torque) produced by a force depends on three factors:

a. The magnitude of the force

b. The distance from the axis to the point of application of the force

c. The angle between the line of action of the force and the line joining the axis to the point of application of the force.

The lever arm (or torque arm) is the perpendicular distance from the axis to the line of action of the force.

The torque τ about an axis perpendicular to the page at point P is defined by either of the following equations (the symbols are defined by the diagram)

$$\tau = FL$$

$$\tau = Fr \sin \theta$$

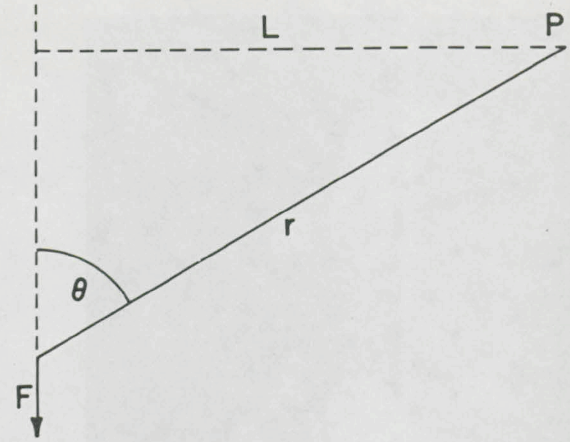


Diagram.

An object is said to be in equilibrium if its linear and rotational velocities are constant. This implies that zero net force and zero net torque act on the object. An object is said to be in static equilibrium if it remains at rest. The conditions for equilibrium are:

a. The net force along any direction is zero.

b. The net torque about any axis is zero.

SECTION B

Elasticity

INTRODUCTION

The elastic properties of the torque wrench will be studied in this section of the module. Instead of using actual torque wrench handles, which often have somewhat complicated shapes, we will analyze uniform rectangular beams of metal. Beams which are held rigidly at one end and which are *loaded* along the length or at the free end are called *cantilever beams*. Thus, in studying the behavior of the torque wrench handle you are also studying a cantilever beam.

EXPERIMENT B-1. The Cantilever Beam

Procedure

1. Set up the apparatus as shown in Figure 20. Adjust the beam to a length of 40 cm, measured from the holder to the end of the beam. Measure the width (B) and the thickness (D) of the beam. Measure the no-load position of the end of the beam by sighting across the top of the beam toward the vertical scale. Record this in the work sheet for this experiment. In each measurement of position, sight across the top of the beam, with your eyes at the beam level. Measure and record the new position of the end of the beam when a mass is hung from the end of the beam. Repeat for several masses over a range of deflections up to about 30 mm. (Some of the beams may be too stiff to get that much deflection with the masses provided.) Subtract the no-load position from each reading to get the value of each deflection.

2. Plot the deflection on the horizontal axis and the weight on the vertical axis. Since the applied force is equal to the weight (mg), this is equivalent to a plot of applied force versus deflection. With a ruler draw the straight line which best fits your data points. Find the slope* of the straight line, and be sure to indicate the units of the slope.

*Pick any two points on the straight line. Measure the *rise*, the vertical distance between those points, and the *run*, the horizontal distance between the same two points. The slope is the rise divided by the run.

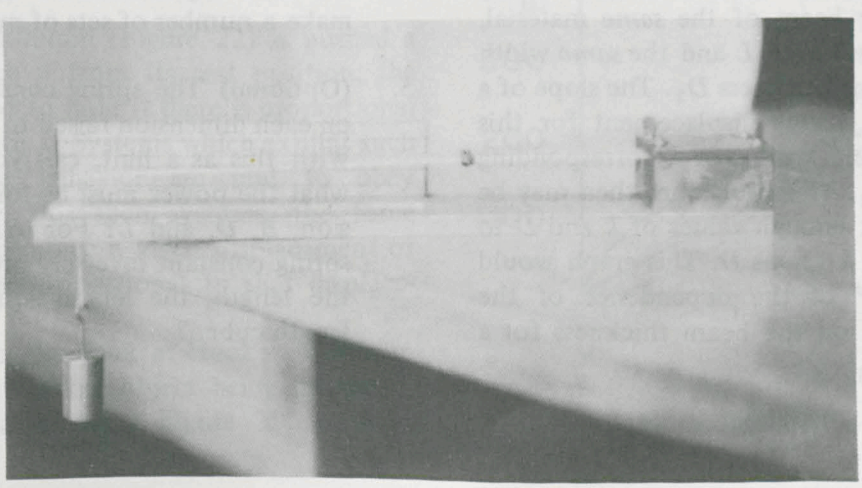


Figure 20.

3. The fact that your data points are very close to a straight line through the origin means that the applied force is proportional to the deflection. Mathematically, we write

$$F = kx$$

The constant of proportionality k is called the *spring constant* or *force constant*. The value of k is equal to the slope of the straight-line graph. What are the units for k? Find it on your graph.

The spring constant is a measure of the "stiffness" of the beam. From experience, you know that the stiffness depends on the size of the beam and on the material of which it is made. For beams of the same material, the stiffness depends on three dimensions, the length L, the thickness D, and the width B.

You can guess, in a general way, how the spring constant depends on each of these three dimensions. For example, does the stiffness increase as the thickness of the beam increases? You could check this by measuring k for beams of the same length and width, but different thicknesses.

The actual procedure would be to choose a beam of length L, width B, and thickness D_1. You would measure the deflection x for various applied forces and plot applied force versus deflection. As before, the slope of this graph gives you a value of the spring constant k corresponding to a set of values of L, B, and D_1. Then you could choose another beam of the *same* material, having the *same* length L and the *same* width B, but a different thickness D_2. The slope of a graph of force versus displacement for this beam would yield a value of k corresponding to thickness D_2. This procedure then may be repeated to get enough values of k and D to plot a graph of k versus D. This graph would be a "picture" of the dependence of the spring constant on the beam thickness for a particular material of a given length and width. You could then make a graph of k versus L and use it to find the dependence of k on length. In a similar manner, the dependence of k on width B could be found.

The dependence of k on length L may be done in the same way, except that you can use the same beam for all of your measurements, only changing the cantilever length.

You can see that, although the procedure is both lengthy and tedious, it is straightforward. After one has found separately the way in which k depends on L, on D, and on B, the dependences can be combined to give a single relation between k and all three quantities.

4. Instead of doing detailed measurements as described, you should try to establish in a general way how k depends on the geometry of the beam. In some cases, you may be able to pool your data with that of other students for different beams of the *same* material. In others, you will have to make additional measurements of k for different beams. You will be able to determine whether k increases or decreases with increasing L, increasing B, or increasing D. For example, two or three values of k corresponding to different values of L will allow you to see whether k increases or decreases as L increases. Several tables have been provided on the work sheet for this experiment so that you can make a number of sets of measurements.

5. (Optional) The spring constant depends on each dimension raised to some power. With this as a hint, can you determine what the power must be for each dimension, B, D, and L? For example, is the spring constant inversely proportional to the length, the length squared, or the length cubed?

HOOKE'S LAW

One of the results of Experiment B-1 is that the deflection x of the end of a cantilever, which is similar to the flexing of a torque wrench handle, is proportional to the applied force F. Mathematically, we write

$$x \propto F$$

which, with the insertion of an appropriate proportionality constant k, becomes

$$F = kx$$

Many other systems behave in the same way. For example, in Figure 21, the force required to stretch a spring a distance x is proportional to x.

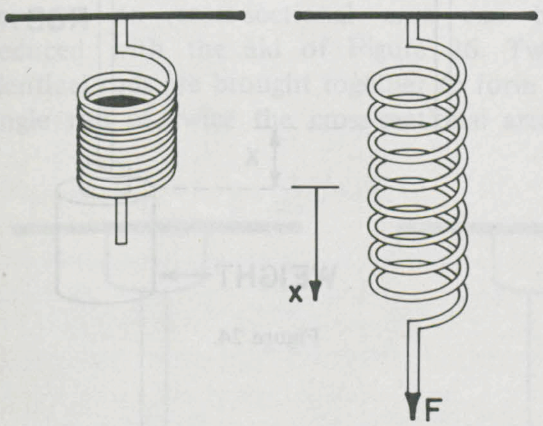


Figure 21.

If a pendulum (Figure 22) is pushed a short distance x from its rest position, the force required to hold it there is proportional to the distance x. Systems which exhibit such force-distance behavior are said to obey *Hooke's law*. For such systems, the force required to produce a given displacement of the system is proportional to that displacement.

One can also look at Hooke's law from the viewpoint of the object being displaced. The stretched spring in Figure 21 exerts an

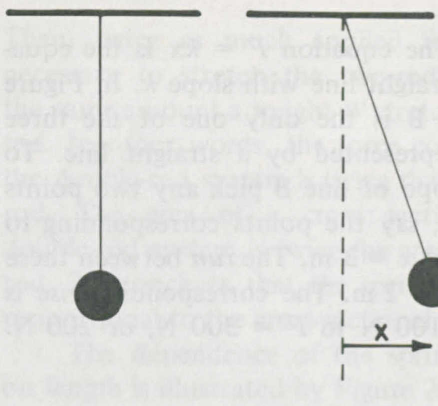


Figure 22.

upward force which is equal and opposite to the applied force F. This upward force, which tries to pull the spring back to its original position, is called a *restoring force*. Since the restoring force is always opposite to the applied force, we can write Hooke's law as

$$F = -kx$$

where the minus sign indicates that the force exerted by the spring always tends to restore the system to its original ($x = 0$) position.

Example 5. Figure 23 shows how the force depends on displacement for three different systems. Which one follows Hooke's law? What is the value of k for that system?

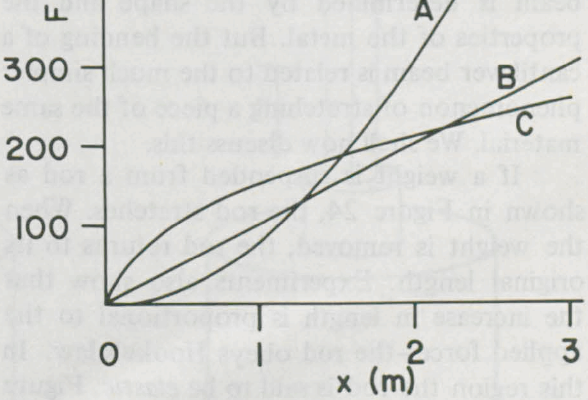


Figure 23.

Solution. The equation $F = kx$ is the equation of a straight line with slope k. In Figure 23, system B is the only one of the three which is represented by a straight line. To find the slope of line B pick any two points on the line, say the points corresponding to $x = 1$ m and $x = 3$ m. The *run* between these two points is 2 m. The corresponding *rise* is from $F = 100$ N to $F = 300$ N, or 200 N. Thus

$$k = \text{slope} = \frac{\text{rise}}{\text{run}}$$

$$= \frac{200\ \text{N}}{2\ \text{m}} = 100\ \text{N/m}$$

Problem 4. Table I is for a spring which follows Hooke's law. Determine the spring constant and fill in the blanks.

Table I.

Displacement (m)	0	0.05	0.1		0.7
Force (N)	0		20	100	

The behavior of many complex systems can be explained by starting from the behavior of simpler systems. For example, the bending of a torque wrench or cantilever beam is determined by the shape and the properties of the metal. But the bending of a cantilever beam is related to the much simpler phenomenon of stretching a piece of the same material. We shall now discuss this.

If a weight is suspended from a rod as shown in Figure 24, the rod stretches. When the weight is removed, the rod returns to its original length. Experiments also show that the increase in length is proportional to the applied force—the rod obeys Hooke's law. In this region the rod is said to be *elastic*. Figure 25 shows the results of such an experiment on a metal rod. The graph exhibits straight-line (Hooke's law) behavior until the force becomes quite large. Then the graph curves. Thus it isn't quite correct to say that the rod always obeys Hooke's law. Rather, it obeys Hooke's law if the applied force isn't too large. This is true for any real material; there is always a limit to the elastic behavior of a real object. When this *elastic limit* is reached, the straight-line behavior changes. Forces much larger than the elastic limit may permanently deform the system. For example, a spring which is stretched too far will not return to its original length. (When this happens, the *yield point* has been reached.) From here on we will restrict our attention to the straight line, or Hooke's law region.

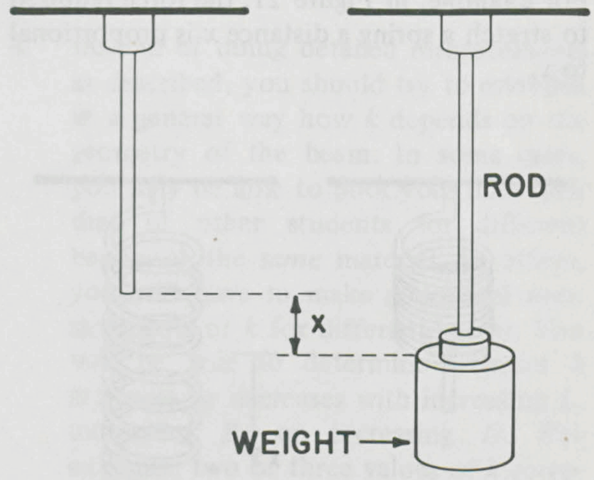


Figure 24.

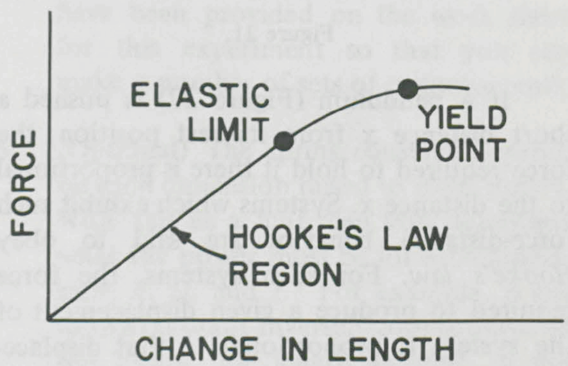


Figure 25.

In the Hooke's law region the behavior of the rod is described by the equation

$$F = kx$$

As with the beam, the constant of proportionality k is called the *spring constant*, or *force constant*. Although the form of this equation is the same as for the bending of the cantilever beam, in this case it refers to stretching instead of bending. For the same rod or beam, the value of k will be different for stretching from the value for bending.

A similar experiment could be performed in which the rod is compressed instead of stretched. Such an experiment shows that, not only is the compression proportional to the applied force, but the spring constant for compressing is the same as for stretching.

The spring constant for stretching depends on the material, the length of the rod, and the area of a cross-section of the rod.

The way in which the spring constant depends on cross-sectional area can be deduced with the aid of Figure 26. Two identical rods are brought together to form a single rod of twice the cross-sectional area. Then, twice as much applied force $2W$ is necessary to stretch the two-rod system by the same amount a weight W stretches a single rod. In other words, the force constant k of the double-rod system is twice that of a single rod. The area of a cross section of the double-rod system is twice the area of a single rod. We conclude that the spring constant is proportional to the cross-sectional area.

The dependence of the spring constant on length is illustrated by Figure 27. A weight W stretches each single rod an amount x. When the two rods are joined end-to-end, each still stretches an amount x so the total stretch is $2x$. (This neglects the weight of the lower rod.) Since the same force produces twice the elongation, the spring constant is half that for the single rod. We conclude that the spring constant is inversely proportional to the unstretched length of the rod. Putting this into mathematical form, we get

$$k \propto A/L$$

where A is the cross-sectional area of the rod

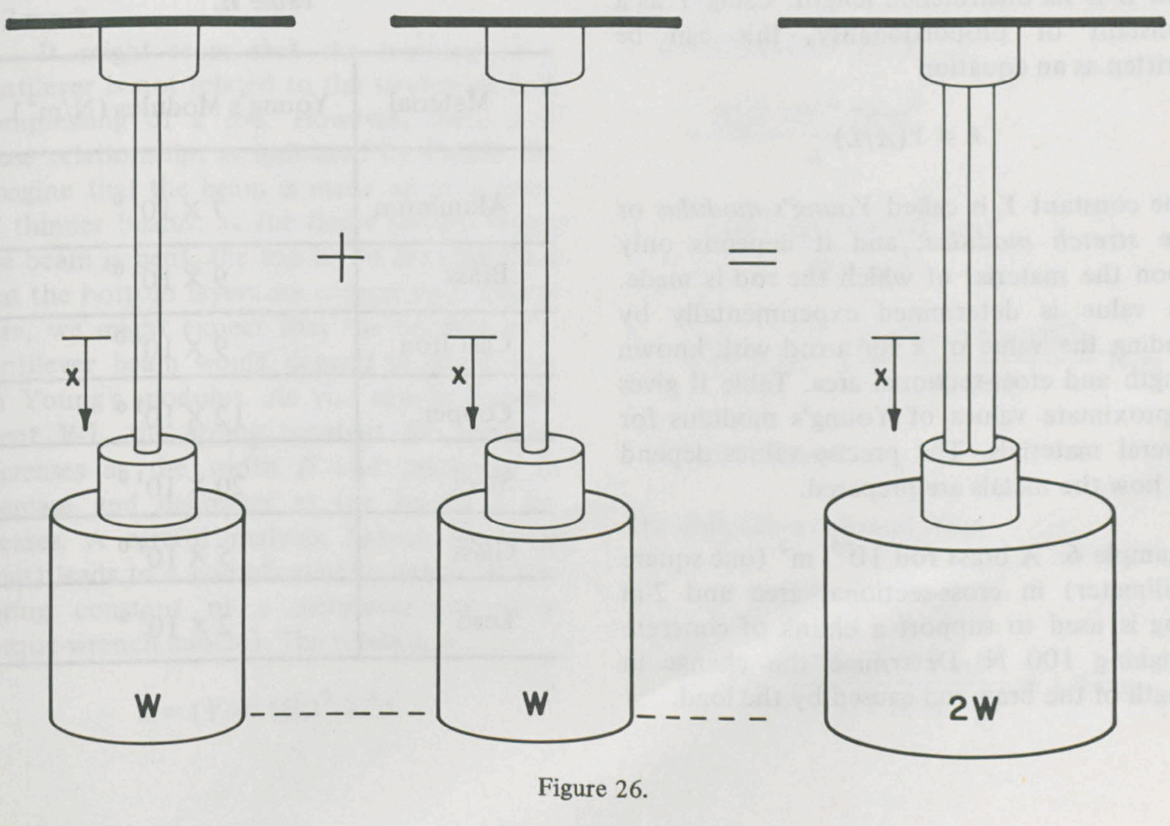


Figure 26.

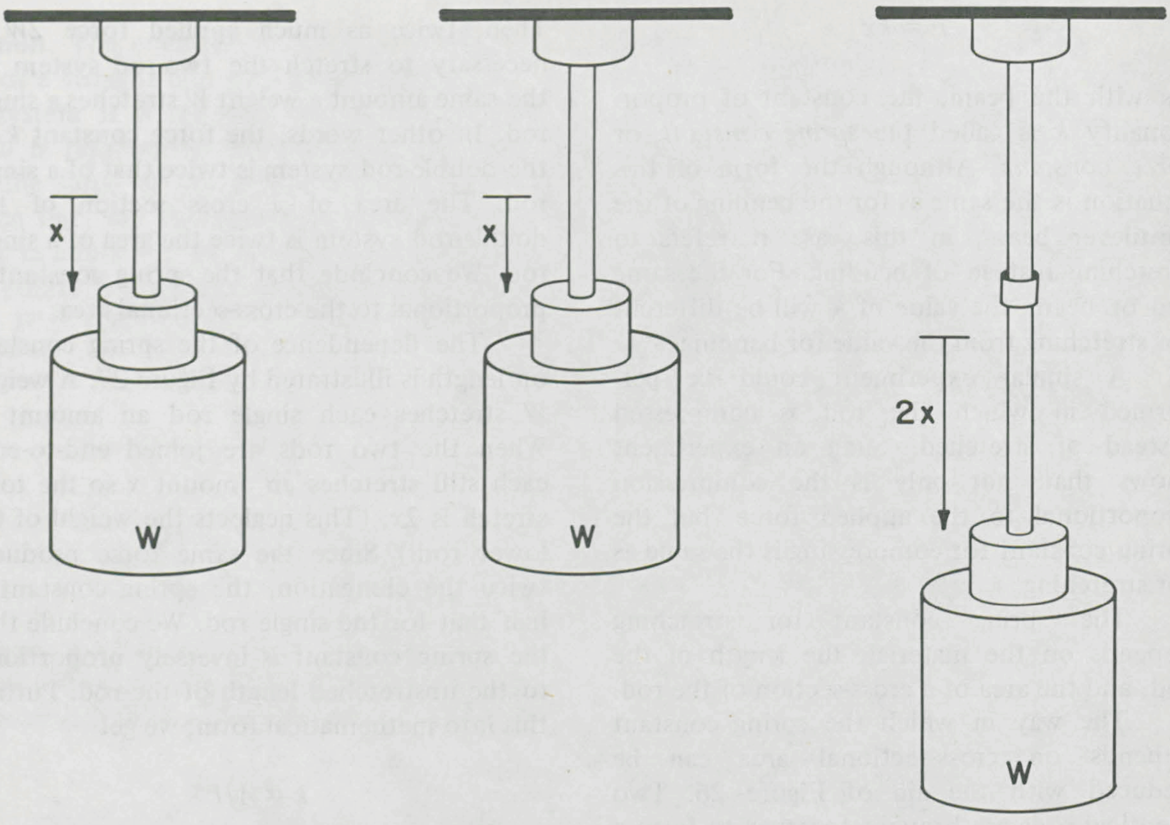


Figure 27.

and L is its unstretched length. Using Y as a constant of proportionality, this can be written as an equation

$$k = Y(A/L)$$

The constant Y is called *Young's modulus* or the *stretch modulus*, and it depends only upon the material of which the rod is made. Its value is determined experimentally by finding the value of k for a rod with known length and cross-sectional area. Table II gives approximate values of Young's modulus for several materials. The precise values depend on how the metals are prepared.

Example 6. A brass rod 10^{-6} m^2 (one square millimeter) in cross-sectional area and 2-m long is used to support a chunk of concrete weighing 100 N. Determine the change in length of the brass rod caused by the load.

Table II.

Material	Young's Modulus (N/m^2)
Aluminum	7×10^{10}
Brass	9×10^{10}
Cast iron	9×10^{10}
Copper	12×10^{10}
Steel	20×10^{10}
Glass	5×10^{10}
Lead	2×10^{10}

Solution.

$$F = kx$$

and

$$k = Y(A/L)$$

Therefore

$$x = F/k = FL/YA$$

Use the data given in the problem together with the value of Young's modulus from Table II:

$$x = \frac{(100\ \text{N})\,(2\ \text{m})}{(9 \times 10^{10}\ \text{N/m}^2)\,(10^{-6}\ \text{m}^2)} =$$

$$2.2 \times 10^{-3}\ \text{m}$$

$$x = 2.2\ \text{mm}$$

Problem 5. A steel rod has a cross-sectional area of 2 mm^2 ($2 \times 10^{-6}\ \text{m}^2$) and a length of 10 m. How much force is required to stretch it 2 mm?

It might seem that the bending of a cantilever is not related to the stretching and compressing of a rod. However, there is a close relationship, as indicated by Figure 28. Imagine that the beam is made up of a stack of thinner beams, as the figure shows. When the beam is bent, the top layers are stretched and the bottom layers are compressed. Therefore, we might expect that the bending of a cantilever beam would depend in some way on Young's modulus. As you saw in Experiment B-1, the spring constant for bending increases as the width B and thickness D increase and decreases as the length L increases. A careful analysis, (which we shall omit) leads to a complicated equation for the spring constant of a cantilever (or of a torque-wrench handle). The relation is:

$$k = (Y/4)\,(BD^3/L^3)$$

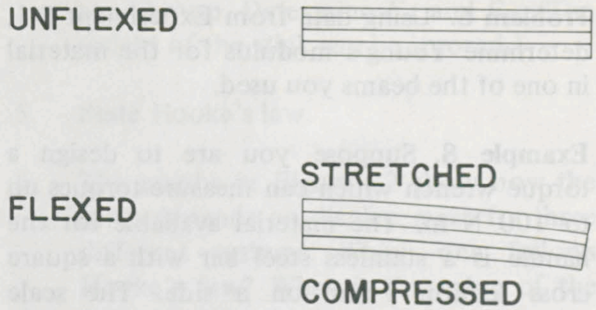


Figure 28.

The appearance of Young's modulus in this expression indicates that the same physical property of the material is important for both stretching and bending.

Example 7. Determine the deflection of the free end of a steel cantilever beam 40 cm long, 1 cm wide, and 3 mm thick. A 0.3-kg mass supplies the loading force.

Solution. First it is necessary to determine the spring constant k of the beam. This requires a straightforward application of the equation for the spring constant for a cantilever beam:

$$k = (Y/4)\,(BD^3/L^3)$$

$$= \frac{20 \times 10^{10}\ \text{N/m}^2}{4} \times$$

$$\frac{(10^{-2}\ \text{m})\,(3 \times 10^{-3}\ \text{m})^3}{(0.4\ \text{m})^3}$$

$$= (5 \times 10^{10})\,\frac{(10^{-2})\,(27 \times 10^{-9})}{(0.064)}\ \text{N/m}$$

$$\cong 211\ \text{N/m}$$

The deflection is found from

$$x = F/k$$

$$x = \frac{0.3\ \text{kg} \times 9.8\ \text{m/s}^2}{211\ \text{N/m}} \cong 1.4\ \text{cm}$$

Problem 6. Using data from Experiment B-1, determine Young's modulus for the material in one of the beams you used.

Example 8. Suppose you are to design a torque wrench which can measure torques up to 100 N·m. The material available for the handle is a stainless steel bar with a square cross section 7 mm on a side. The scale available is 100 mm long (50 mm on each side of zero). How long should the handle be?

Solution. Assume that the cantilever relation is valid:

$$F = kx = (Y/4)\,(BD^3/L^3)\,x$$

Also assume that the force is applied to the end of the handle, as indicated in Figure 29.

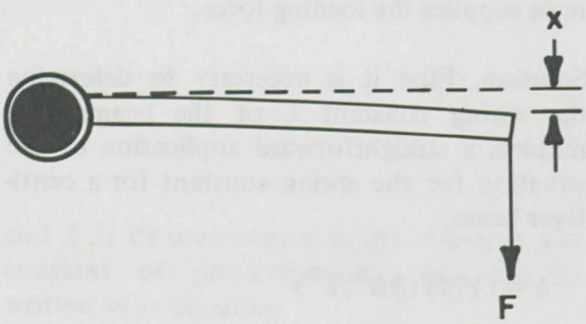


Figure 29.

Then the torque is

$$\tau = FL$$

$$= (Y/4)\,(BD^3/L^3)\,xL$$

$$= (Y/4)\,(BD^3/L^2)\,x$$

Then

$$L^2 = (Y/4)\,BD^3 x/\tau$$

For the maximum deflection, this is

$$L^2 = \frac{(20 \times 10^{10}\ \mathrm{N/m^2})\,(7 \times 10^{-3}\ \mathrm{m^2})^4}{4 \times 100\ \mathrm{N\cdot m}} \times$$

$$(50 \times 10^{-3}\ \mathrm{m})$$

$$\cong .06\ \mathrm{m^2}$$

$$L \cong .245\ \mathrm{m} = 24.5\ \mathrm{cm}$$

The handle of the wrench should be about 24.5 cm long.

Problem 7. A small torque wrench is designed to measure torques up to 25 in·lb. The part of the handle which bends (the cantilever) is 3.8 in long and it has a square cross section 0.1 in on a side. The deflection of the end of the cantilever at maximum torque is 0.5 in. Find Young's modulus for the material from which the handle is made. (To convert your answer from $\mathrm{lb/in^2}$ to $\mathrm{N/m^2}$, multiply it by 6.9×10^3.)

SUMMARY

Beams which are held rigidly at one end and loaded along the length or at the other end are called *cantilever beams*.

Many systems, including the cantilever beam, obey *Hooke's law*: the force required to produce a displacement is proportional to the displacement. That is, $F = kx$. The constant of proportionality k is called the *spring constant*.

The spring constant k for a stretched (or compressed) rod depends on the cross-sectional area A, the unstretched length L, and a property of the material called *Young's modulus, Y*:

$$k = Y(A/L)$$

The spring constant k for a cantilever beam depends on the length L, width B, thickness D, of the beam, and on Young's modulus. $k = (Y/4)\,(BD^3/L^3)$.

The torque produced by exerting a force F on the end of the cantilever is

$$\tau = (Y/4)\,(BD^3/L^2)\,x$$

where x is the deflection of the end of the beam.

ADDITIONAL PROBLEMS

1. Define the following:

   a. Torque wrench

   b. Torque

   c. Lever arm

   d. Equilibrium

   e. Static equilibrium

   f. Spring constant

   g. Cantilever beam

2. State the conditions which the applied forces and torques must satisfy for an object to be in equilibrium.

3. For the situation shown in Figure 30, determine the lever arm and the torque associated with the force F. Use the point P for the axis.

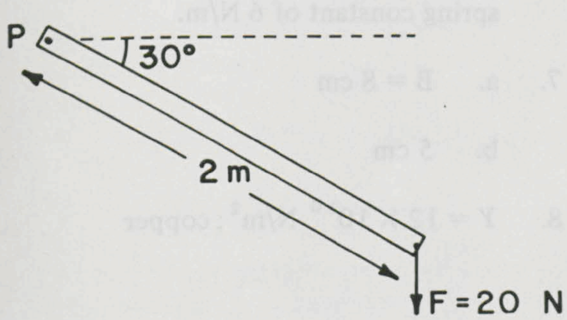


Figure 30.

4. The stick shown in Figure 31 is in equilibrium. Determine F_2 and F_3. (The weight of the stick can be ignored.)

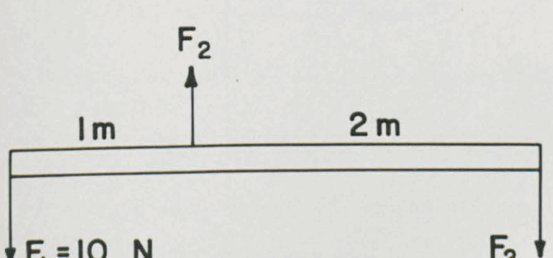


Figure 31.

5. State Hooke's law.

6. The graphs in Figure 32 show how the force depends on displacement for three different systems. Which one follows Hooke's law? What is the value of the spring constant?

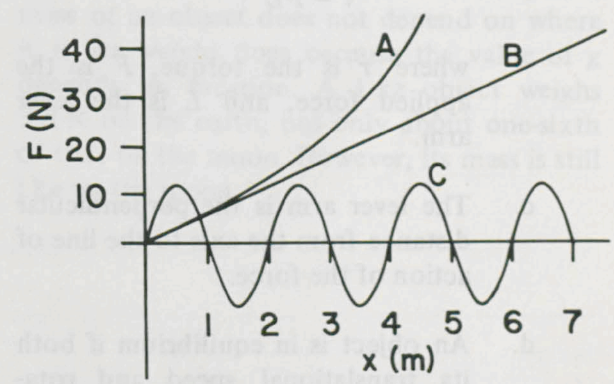


Figure 32.

7. The equation for the spring constant for a cantilever is

$$k = (Y/4)\,(BD^3/L^3)$$

   a. If a steel cantilever 2 m long and 1 cm thick is to have a spring constant of 500 N/m, how wide must it be? (Young's modulus for steel is 20×10^{10} N/m^2.)

   b. How much will the end deflect if a force of 25 N is applied to the end, at a right angle to the beam?

8. A rod 2 m long has a cross-sectional area of 4 mm^2. To stretch it 3 mm, a force of 720 N is required. Determine Young's modulus for the material. If the material is one of those listed in Table II, which must it be?

Answers for Additional Problems

1. Definitions:

   a. A torque wrench is a tool which can apply a torque to a nut or bolt and which has a scale to indicate how much torque is being applied.

   b. Torque is the twisting effect produced by an applied force. It is defined by the equation

   $$\tau = FL$$

   where τ is the torque, F is the applied force, and L is the lever arm.

   c. The lever arm is the perpendicular distance from the axis to the line of action of the force.

   d. An object is in equilibrium if both its translational speed and rotational speed are constant.

   e. An object which is not moving is in static equilibrium.

   f. For systems which obey Hooke's law, the force required to produce a displacement is proportional to the displacement. The constant of proportionality is called the spring constant k.

   g. Beams which are held rigidly at one end and loaded along the length or at the other end are called cantilever beams.

2. For an object to be in equilibrium, two conditions must be satisfied:

   a. The net force must be zero.

   b. The net torque about any axis must be zero.

3. Lever arm = 1.73 m

   Torque = 34.6 N·m

4. $F_3 = 5$ N

   $F_2 = 15$ N

5. For systems which obey Hooke's law, the force required to produce a displacement is proportional to the displacement.

6. System B obeys Hooke's law, and has a spring constant of 6 N/m.

7. a. B = 8 cm

   b. 5 cm

8. $Y = 12 \times 10^{10}$ N/m^2; copper

APPENDIX

Weight and Mass

Weight and mass are different concepts. The *weight* of an object is the gravitational force acting on it. For example, if you hold a piece of iron in your hand you can feel its weight, which is due to the gravitational attraction which the earth exerts on the iron.

The *mass* of an object is a measure of its resistance to any change in its motion. You feel this property when you throw an object. You must exert a force on the object to change its motion from a state of rest to one in which it is moving. Similarly, you feel it when you stop a moving object; for example, when you catch a baseball.

The weight of any object is proportional to its mass. If we let W represent the weight and M the mass, then

$$W = Mg$$

where g is the acceleration due to gravity, if the object were freely falling. The value of g near the surface of the earth is about $9.8\ m/s^2$. Thus a 10-kg object has a weight of 98 N.

The relation between weight and mass emphasizes an important point. Although the mass of an object does not depend on where it is, its weight does because the value of g depends on location. A 1-kg object weighs 9.8 N on the earth, but only about one-sixth of that on the moon. However, its mass is still 1 kg on the moon.

WORKSHEET

Experiment A-1. Torque

A. Changing the Amount of the Hanging Mass

Mass (g)	Torque (in·lb)

B. Changing the Position of the Hanging Mass

Mass = ________ g

Distance (d)	Torque (in·lb)

C. Changing the Angle of the Lever Arm

Lever Arm (L)	Torque (in·lb)

WORKSHEET

Experiment B-1. The Cantilever Beam

1. Beam width B = __________

Thickness D = __________

Length L = __________

Suspended Mass (kg)	Suspended Weight (N)	No-load Position (mm)	Loaded Position (mm)	Deflection (mm)

Spring constant k = __________ N/m

2. Beam width B = __________

Thickness D = __________

Length L = __________

Suspended Mass (kg)	Suspended Weight (N)	No-load Position (mm)	Loaded Position (mm)	Deflection (mm)

Spring constant k = __________ N/m

3. Beam width B = __________

Thickness D = __________

Length L = __________

Suspended Mass (kg)	Suspended Weight (N)	No-load Position (mm)	Loaded Position (mm)	Deflection (mm)

Spring constant k = __________ N/m

Part 4

Does the stiffness (spring constant k) of the cantilever beam increase, decrease, or stay the same when

a. The length L is increased? __________

b. The thickness D is increased? __________

c. The width B is increased? __________

Write an equation for the spring constant in the form: $k = (\text{a number})\ L^n B^m D^p$ where n, m and p are some small whole numbers.

3. Beam width B = __________

Thickness D = __________

Length L = __________

Suspended Mass (kg)	Suspended Weight (N)	Unloaded Position (mm)	Loaded Position (mm)	Deflection (mm)

Spring constant k = __________ N/m

Part 4

Does the stiffness (spring constant k) of the cantilever beam increase, decrease, or stay the same when

a. The length L is increased?

b. The thickness D is increased?

c. The width B is increased?

Write an equation for the spring constant in the form $k = (\text{constant})\, L^a B^b D^c$ where a, b and c are small whole numbers.

0-07-001725-5